Short Course on Experimental Dynamic Substructuring

Module #6: Transmission Simulator Method

M. S. Allen, R. L. Mayes, and E. J. Bergman, "Experimental Modal Substructuring to Couple and Uncouple Substructures with Flexible Fixtures and Multi-point Connections," Journal of Sound and Vibration, vol. 329, pp. 4891–4906, 2010, http://dx.doi.org/10.1016/j.jsv.2010.06.007.

Matthew S. Allen
Associate Professor, University of Wisconsin-Madison
Daniel Rixen
Chair, Institute of Applied Mechanics, Technische Universität München
Randall L. Mayes
Distinguished Member of Technical Staff, Sandia National Labs.

Short Course Notes For:
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Modal Substructuring (CMS)

- As discussed previously, if we have the modal parameters of two systems, either from analysis or experiment, we can write:

$$\begin{bmatrix} \mathbf{I}^{\mathrm{A}} & 0 \\ 0 & \mathbf{I}^{\mathrm{B}} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{q}}^{\mathrm{A}} \\ \ddot{\mathbf{q}}^{\mathrm{B}} \end{Bmatrix} + \begin{bmatrix} \left\lceil 2\zeta_r\omega_r \right\rfloor^{\mathrm{A}} & \mathbf{0} \\ \mathbf{0} & \left\lceil 2\zeta_r\omega_r \right\rfloor^{\mathrm{B}} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{q}}^{\mathrm{A}} \\ \dot{\mathbf{q}}^{\mathrm{B}} \end{Bmatrix} + \begin{bmatrix} \left\lceil \omega_r^2 \right\rfloor^{\mathrm{A}} & \mathbf{0} \\ \mathbf{0} & \left\lceil \omega_r^2 \right\rfloor^{\mathrm{B}} \end{bmatrix} \begin{Bmatrix} \mathbf{q}^{\mathrm{A}} \\ \mathbf{q}^{\mathrm{B}} \end{Bmatrix} =$$

$$= \begin{bmatrix} (\mathbf{\Phi}^{\mathrm{A}})^{\mathrm{T}} & \mathbf{0} \\ \mathbf{0} & (\mathbf{\Phi}^{\mathrm{B}})^{\mathrm{T}} \end{bmatrix} \begin{Bmatrix} \mathbf{F}^{\mathrm{A}} \\ \mathbf{F}^{\mathrm{B}} \end{Bmatrix}$$

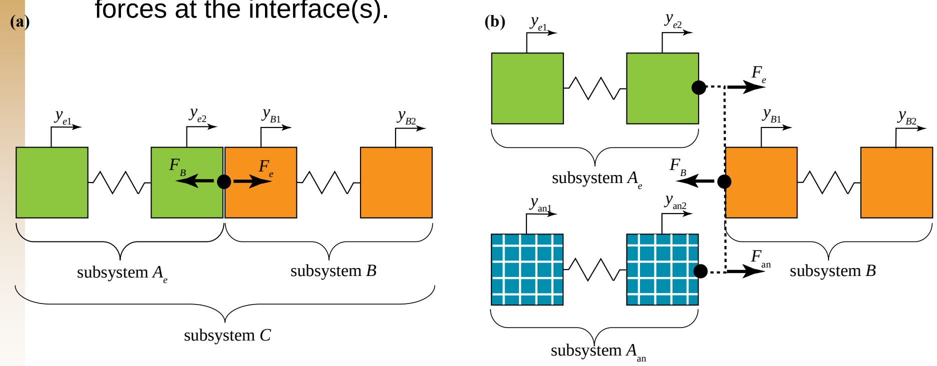
$$\begin{Bmatrix} \mathbf{y}^{\mathrm{A}} \\ \mathbf{y}^{\mathrm{B}} \end{Bmatrix} = \begin{bmatrix} \mathbf{\Phi}^{\mathrm{A}} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi}^{\mathrm{B}} \end{bmatrix} \begin{Bmatrix} \mathbf{q}^{\mathrm{A}} \\ \mathbf{q}^{\mathrm{B}} \end{Bmatrix}$$

- Then, constraints in the following form are enforced to couple the substructures:

$$\mathbf{B}_{\mathrm{p}} \begin{Bmatrix} \mathbf{y}^{\mathrm{A}} \\ \mathbf{y}^{\mathrm{B}} \end{Bmatrix} = \mathbf{0} \qquad \mathbf{B} = \mathbf{B}_{\mathrm{p}} \begin{bmatrix} \mathbf{\Phi}^{\mathrm{A}} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi}^{\mathrm{B}} \end{bmatrix} \qquad \mathbf{B} \begin{Bmatrix} \mathbf{q}^{\mathrm{A}} \\ \mathbf{q}^{\mathrm{B}} \end{Bmatrix} = \mathbf{0}$$

Substructure Uncoupling

- Our approach to “uncouple” structures works by constraining a (negative) model of one component to the structure that cancels the forces at the interface(s).



$$\mathbf{I}_e\ddot{\mathbf{q}}_e + \left[{}^{\backslash}2\zeta_r\omega_{r\backslash}\right]_e\dot{\mathbf{q}}_e + \left[{}^{\backslash}\omega_{r\backslash}^2\right]_e\mathbf{q}_e = \mathbf{\Phi}_e{}^{\mathrm{T}}\mathbf{F}_e$$

$$+\quad \mathbf{I}_{an}\ddot{\mathbf{q}}_{an} + \left[{}^{\backslash}2\zeta_r\omega_{r\backslash}\right]_{an}\dot{\mathbf{q}}_{an} + \left[{}^{\backslash}\omega_{r\backslash}^2\right]_{an}\mathbf{q}_{an} = \mathbf{\Phi}_{an}{}^{\mathrm{T}}\mathbf{F}_{an}$$

Forces exerted by A and (-A) onto B

0

The interfaces forces can be made to cancel if:

$$\mathbf{I}_e\ddot{\mathbf{q}}_e + \mathbf{I}_{an}\ddot{\mathbf{q}}_{an} + \left[{}^{\backslash}2\zeta_r\omega_{r\backslash}\right]_e\dot{\mathbf{q}}_e + \left[{}^{\backslash}2\zeta_r\omega_{r\backslash}\right]_{an}\dot{\mathbf{q}}_{an} + \left[{}^{\backslash}\omega_{r\backslash}^2\right]_e\mathbf{q}_e + \left[{}^{\backslash}\omega_{r\backslash}^2\right]_{an}\mathbf{q}_{an} = \\ = \mathbf{\Phi}_e{}^{\mathrm{T}}\mathbf{F}_e + \mathbf{\Phi}_{an}{}^{\mathrm{T}}\mathbf{F}_{an}$$

1.) The motion of both the experimental and analytical fixtures is the same, $\{\mathbf{q}_{an}(t)\} = \{\mathbf{q}_e(t)\}$

$$\left[\mathbf{I}_e + \mathbf{I}_{an}\right](\ddot{\mathbf{q}}_e = \ddot{\mathbf{q}}_{an}) + \left[\left[{}^{\backslash}2\zeta_r\omega_{r\backslash}\right]_e + \left[{}^{\backslash}2\zeta_r\omega_{r\backslash}\right]_{an}\right](\dot{\mathbf{q}}_e) + \left[\left[{}^{\backslash}\omega_{r\backslash}^2\right]_e + \left[{}^{\backslash}\omega_{r\backslash}^2\right]_{an}\right]\mathbf{q}_e = \\ = \mathbf{\Phi}_e{}^{\mathrm{T}}\mathbf{F}_e + \mathbf{\Phi}_{an}{}^{\mathrm{T}}\mathbf{F}_{an}$$

2.) The modal parameters of the analytical and true fixtures are equal and opposite, i.e. $\left[{}^{\backslash}\omega_{r\backslash}^2\right]_{an} = -\left[{}^{\backslash}\omega_{r\backslash}^2\right]_e$ and similarly for the mass and damping terms.

The interfaces forces can be made to cancel if:

$$\left[\mathbf{I}_e - \mathbf{I}_{\text{an}}\right]\left(\ddot{\mathbf{q}}_e = \ddot{\mathbf{q}}_{\text{an}}\right) + \left[\left[{}^{\backslash}2\zeta_r\omega_{r\backslash}\right]_e - \left[{}^{\backslash}2\zeta_r\omega_{r\backslash}\right]_{\text{an}}\right]\left(\dot{\mathbf{q}}_e\right) + \left[\left[{}^{\backslash}\omega_{r\backslash}^2\right]_e - \left[{}^{\backslash}\omega_{r\backslash}^2\right]_{\text{an}}\right]\mathbf{q}_{\text{e}} =$$
$$= \mathbf{\Phi}_e{}^{\text{T}}\mathbf{F}_e + \mathbf{\Phi}_{\text{an}}{}^{\text{T}}\mathbf{F}_{\text{an}}$$

3.) The mode shapes of both the experimental and analytical models are equal $\mathbf{\Phi}_{\text{an}} = \mathbf{\Phi}_e$.

4.) The interface force vector is in the range space of $\mathbf{\Phi}_{\text{an}}{}^{\text{T}}$.

$$\left[\sim 0\right]\left(\ddot{\mathbf{q}}_e = \ddot{\mathbf{q}}_{\text{an}}\right) + \left[\sim 0\right]\left(\dot{\mathbf{q}}_e\right) + \left[\sim 0\right]\mathbf{q}_{\text{e}} = \mathbf{\Phi}_{\text{an}}{}^{\text{T}}\left(\mathbf{F}_e + \mathbf{F}_{\text{an}}\right)$$

Connection Methods – CPT and MCFS

- One cannot measure the response at the connection point directly, so it must be estimated from other measurements.
  - CPT Method: Connection point responses for the experimental system are estimated using a modal filter and constrained to the analytical fixture A and beam D:

$$\begin{Bmatrix} \left\{\mathbf{y}^{\text{C}}\right\}_m \\ \left\{\mathbf{y}^{\text{C}}\right\}_c \end{Bmatrix} \approx \begin{bmatrix} \left[\mathbf{\Phi}^{\text{A}}{}_m\right] \\ \left[\mathbf{\Phi}^{\text{A}}{}_c\right] \end{bmatrix} \left\{\mathbf{q}^{\text{C}}\right\} \quad \rightarrow \quad \left\{\mathbf{y}^{\text{C}}\right\}_c = \left[\mathbf{\Phi}^{\text{A}}{}_c\right]\left[\mathbf{\Phi}^{\text{A}}{}_m\right]^{\dagger}\left\{\mathbf{y}^{\text{C}}\right\}_m$$

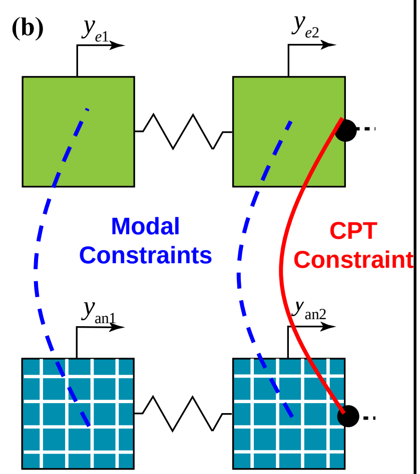
  - Modal Constraint Method: Constrain the modal DOF of the Fixture model to their approximation on C: (aka MCFS or “Modal Constraint for Fixture and Subsystem”)

$$\left.\begin{aligned} \left\{\mathbf{q}^{\text{A}}\right\} &= \left[\mathbf{\Phi}^{\text{A}}{}_m\right]^{\dagger}\left\{\mathbf{y}^{\text{A}}\right\}_m \\ \left\{\mathbf{q}^{\text{C}}\right\} &= \left[\mathbf{\Phi}^{\text{A}}{}_m\right]^{\dagger}\left\{\mathbf{y}^{\text{C}}\right\}_m \end{aligned}\right\} \quad \rightarrow \quad \left\{\mathbf{q}^{\text{A}}\right\} = \left\{\mathbf{q}^{\text{C}}\right\}$$

Note, these are not the modes of C!

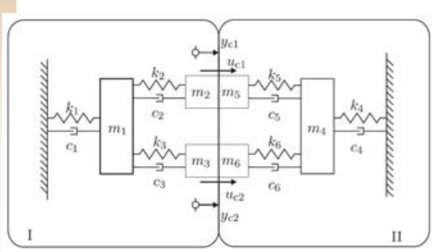
Connection Methods – Rationalization

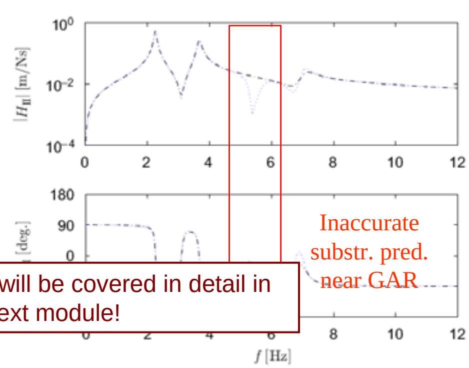
- CMS can be very sensitive to errors when removing a substructure from a system.
- If the fixture is elastic, then it will probably have more modal DOF (N_A) than there are connection point DOF (N_c).
  - e.g. requiring equal 2D motion at the connection point enforces only (3) constraints.
  - When the number of constraints is $N_c<N_A$, it is unlikely that the modes of both structures will match and the interface forces most likely will not cancel!



Related Techniques

- Sjovall & Abrahamsson observed a similar phenomenon using FBS and called it “generalized anti-resonance” (GAR).
- Suggested using internal DOF to eliminate motions that are poorly observable on the interface DOF.
  - *W. D’Ambrogio and A. Fregolent, (IMAC XXVII) Orlando, Florida, 2009.*
  - *Sjovall, P. and T. Abrahamsson (2008). MSSP **22**(1): 15-33.*

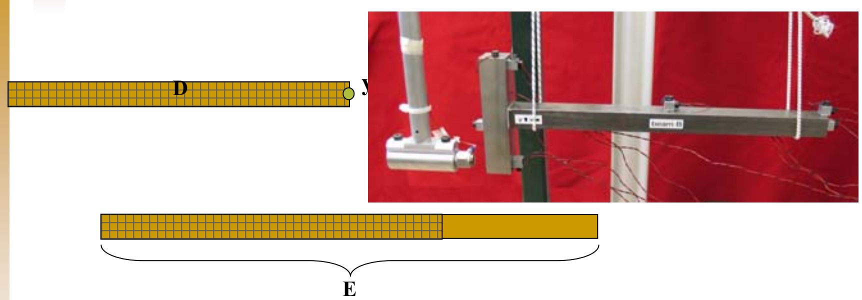




$H(\omega_{\mathrm{GAR}})U$

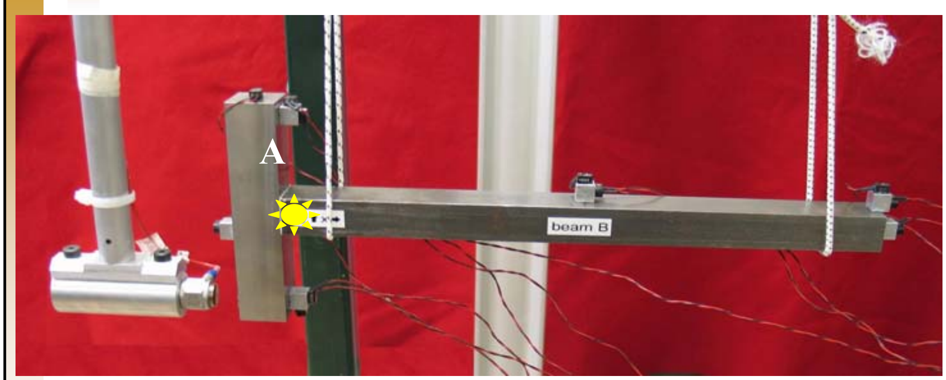
These approaches will be covered in detail in the next module!

Test Case #1: T-Beam



- Objective: Join an experimental model of beam B to an analytical model for beam D at point y_c.
  - Measurements on system C = beam B + fixture A
  - Analytical model (Euler-Bernoulli) for fixture A removed from C
  - Beam B + Beam D (tuned Euler-Bernoulli analytical model) = assembly Beam E

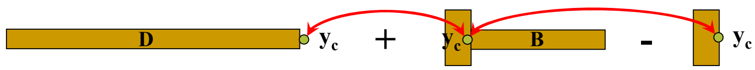
Experimental Procedure



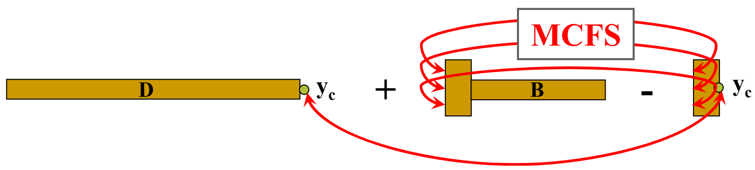
- Careful tests used to estimate the modal parameters of the C system at a number of points on fixture A.
- Hammer used to excite the largest bandwidth (largest number of modes) possible and to avoid modifying structure with the exciter.

Cases Considered

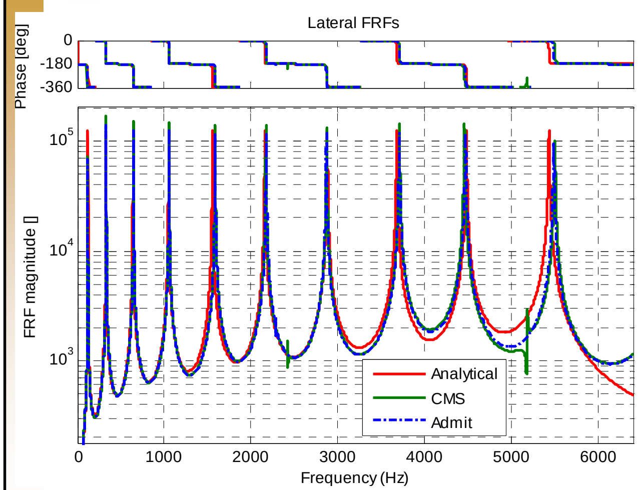
- Case 1: **CPT**: Models for A, C and D joined at the connection point.
  - Case 1a: Rigid Fixture Model
  - Case 1b: Elastic Fixture Model



- Case 2: **MCFS**: Models for A and C joined using MCFS method.
  - Elastic Fixture Model



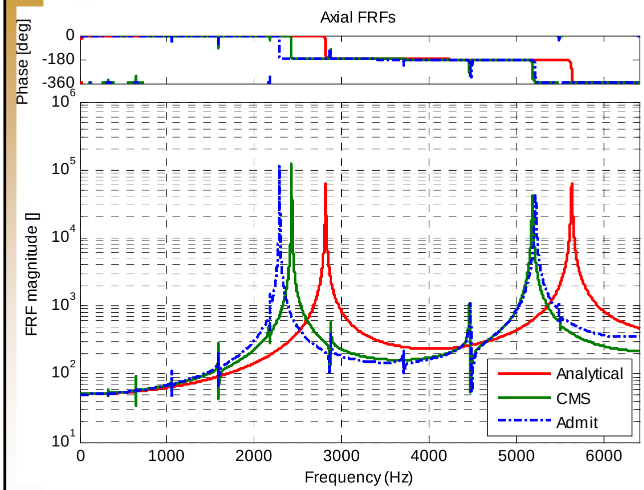
Case 1a: Rigid A, CPT



"Admit" = FBS with reconstructed FRFs

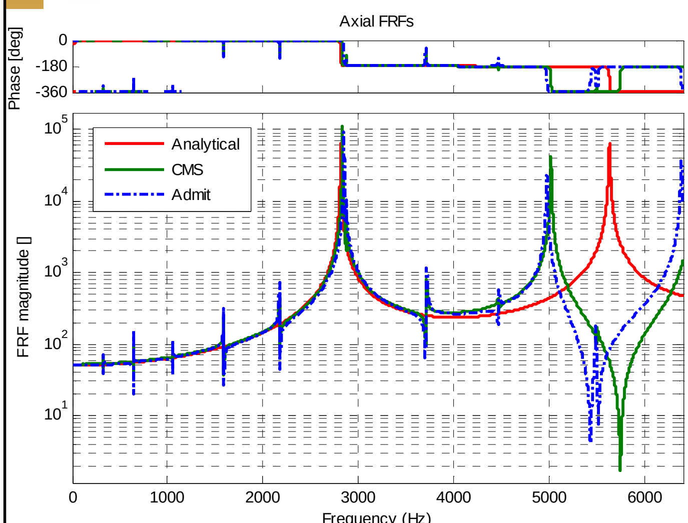
- Excellent results obtained in the lateral direction (Bending Modes).
- Both the CMS and FRF based Admittance procedures agree very well with the analytical model.
- The CMS result is slightly contaminated by the axial modes at 2400 and 5200 Hz.

Case 1a: Rigid A, CPT



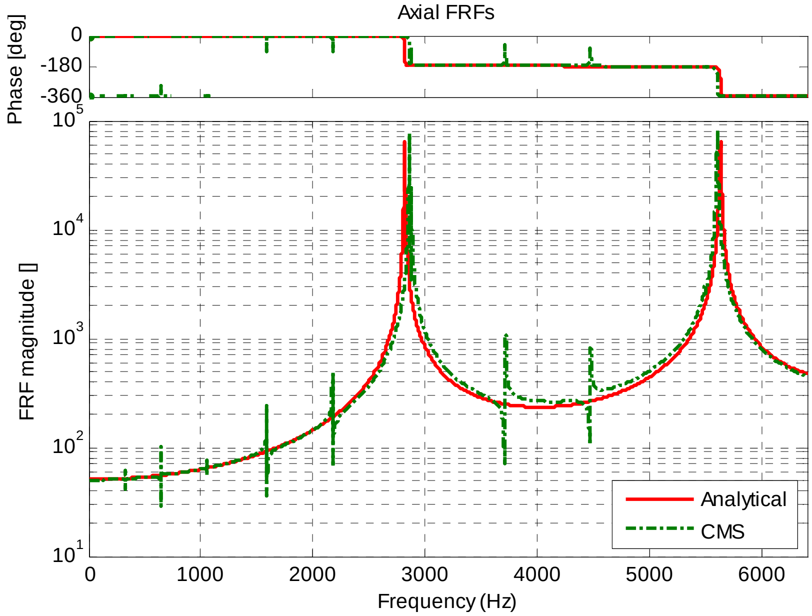
- Admittance and CMS both under-predict the natural frequencies of the axial modes by 10% or more.
  - Fixture flexibility is important!
- Contamination at the bending natural frequencies.
  - Possibly due to small curve fitting errors or cross axis sensitivity.

Case 1b: Flexible A, CPT

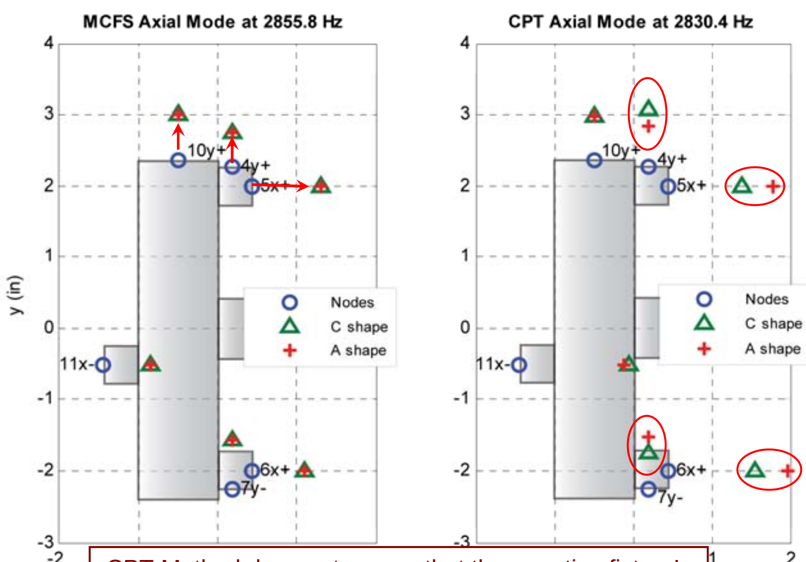


- Both FBS and CMS accurately predict the first axial frequency with the flexible fixture model.
- Both methods more severely under-predict the second axial mode.
- Both predict a spurious zero near 5500 Hz.
- The Lateral FRFs were similar to those shown previously.

Case 2: Flexible A, MCFS Method



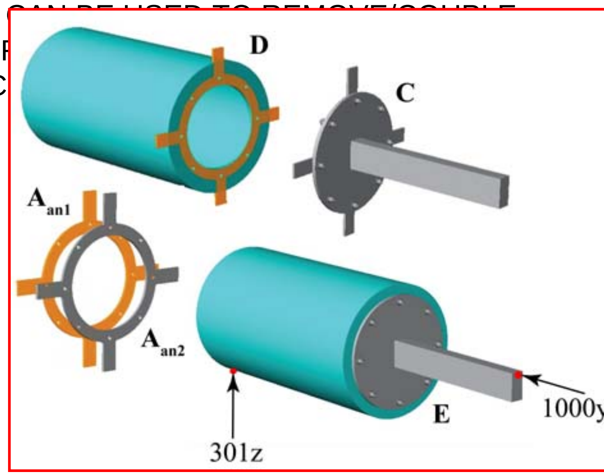
Explanation:



CPT Method does not assure that the negative fixture's motion matches the motion of the true fixture!

Significance:

- Now it is possible to robustly remove one flexible structure from another.
- The fixture modal basis gives a convenient basis for the interface – STRUCTUR US INTERFAC



Transmission Simulator Method Enables use of Continuous Interfaces

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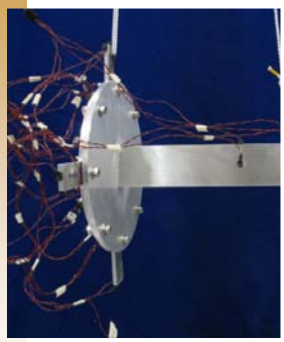
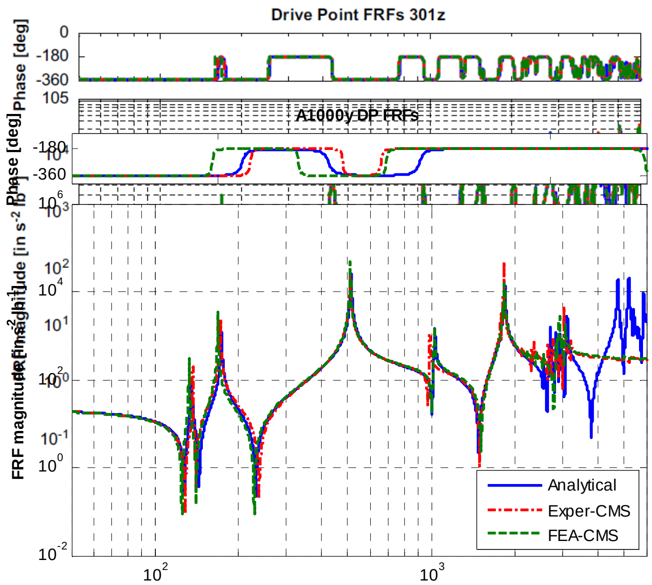




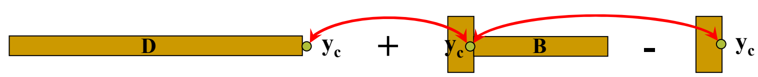
Transmission Simulator Design

- Design the TS so that it can be instrumented to capture all of its relevant dynamic modes so that $[\Phi_m]$ will be well conditioned for inversion.
- Choose the mass of the fixture to bring more modes into the testable bandwidth.
- Design the TS so that it can be modeled accurately. Avoid designs that contain joints, intricate geometry, or other features that are difficult to accurately model analytically.
- The joint between the fixture and the test article should replicate the actual joint in the system of interest as closely as possible, to capture the joint stiffness and damping.
- The fixture's impedance should roughly resemble that of the built-up structure. The fixture might just be a chopped off version of the other substructure which precisely mimics the mass and stiffness near the connection interface.

Problem of Negative Mass (& Stiffness)

- The transmission simulator must be removed to obtain the desired experimental model.
- The subtraction process may introduce negative mass and stiffness.
- Models with negative mass and stiffness are not readily imported into FEA packages, and they may introduce errors or counter-intuitive results.
- Example: Beam problem mentioned earlier

	Complex f_n (Hz)	eig(M) < 0
Case 1a Rigid Fixture, Con. Pt.	0 + i*8.9e-5 0 + i*5.5e5	-0.011
Case 1b Flexible Fixture, Con. Pt.	0 + i*1.36e-4 8951 – i*2450 8951 + i*2450	-1
Case 2 Flexible Fixture, MCFS	0 + i*2.28e-4 13050 – i*4285 13050 + i*4285	-0.086



Modal Scale Factor Method (Mayes)

- Negative mass implies that too much mass is being subtracted $\rightarrow$ the scale factors of the transmission simulator modes may be in error.
  - Solution: Adjust the modal scale factors to produce a positive-definite mass matrix. But how?
- Eigenvalues of Mass Matrix of B (structure of interest)

$$\mathbf{M}_{\mathrm{B}} \boldsymbol{\psi}_k = \lambda_k \boldsymbol{\psi}_k \qquad \lambda_k < 0 \quad k = 0 \ldots N_{\mathrm{neg}}$$

- There are typically only a few negative eigenvalues and several modal scale factors (mm_j) that could be modified $\rightarrow$ under-constrained optimization problem.

$$\mathbf{f}\left(\mathrm{mm}_j\right) = \begin{bmatrix} \lambda_k & \cdots & \lambda_{N_{\mathrm{neg}}} \end{bmatrix}^{\mathrm{T}}$$

$$\mathbf{f}(\mathrm{mm}_j + \Delta\mathrm{mm}_j) = \mathbf{f}(\mathrm{mm}_j) + \left[\nabla \mathbf{f}(\mathrm{mm}_j)\right]_{\mathrm{mm}_j} \Delta\mathrm{mm}_j$$

Added Mass Method (Allen)

- Negative mass can be eliminated by adding mass to the structure until the total mass is positive.
  - Suppose point masses are added at any of the points where the system's modes were measured. This changes the EOM by adding mass at each point (or each mode if using modal coord.): $\mathbf{M}_{\mathrm{B}} + \Delta \mathbf{m}$
  - The smallest change to the mass matrix that will make all of its eigenvalues positive is:

$$\Delta\lambda_k = \begin{cases} 0 & \lambda_k > 0 \\ -\lambda_k + \varepsilon & \lambda_k \leq 0 \end{cases} \qquad \mathbf{M}_{\mathrm{B}} \boldsymbol{\psi}_k = \lambda_k \boldsymbol{\psi}_k$$

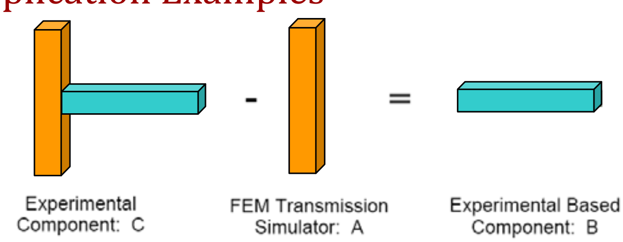
These are eigenvectors / eigenvalues of the mass matrix, not modes of B.

  - where ε is a small value. This can be obtained using:

$$\Delta \mathbf{m} = \boldsymbol{\psi} \hat{\boldsymbol{\Lambda}} \boldsymbol{\psi}^{\mathrm{T}} \qquad \boldsymbol{\psi} = \begin{bmatrix} \boldsymbol{\psi}_k & \cdots & \boldsymbol{\psi}_{N_C} \end{bmatrix}$$

$$\hat{\boldsymbol{\Lambda}} = diag\left[\left(\lambda_1 + \Delta\lambda_1\right) \quad \cdots \quad \left(\lambda_{Nc} + \Delta\lambda_{N_C}\right) \right]$$

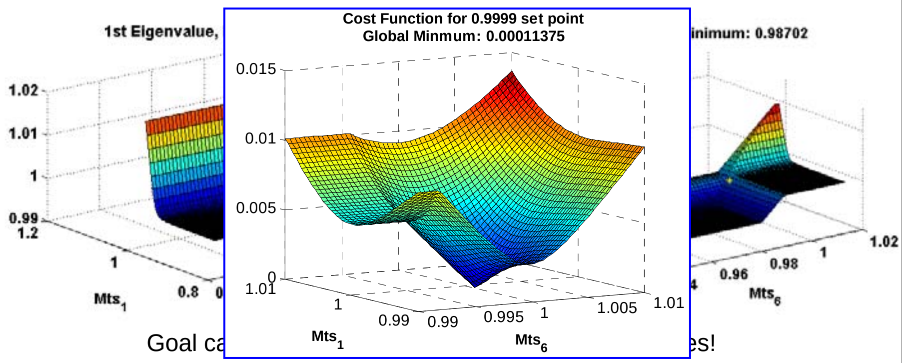
Application Examples



- Modal test obtains all modes below 20kHz for C and below 24kHz for A $\rightarrow$ 7 modes for TS and 15 modes for C
- The mass matrix of the resulting model for B has two negative eigenvalues:
  - λ_1= -0.00050468
  - λ_2=-2.4058e-16, (essentially zero).
  - We desire to make these eigenvalues positive using the proposed methods.

Results: Modal Scale Factor Method

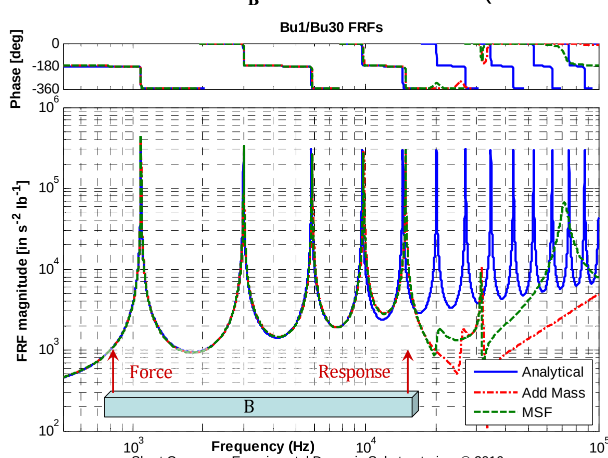
- Jacobian (or metrics) reveal that the negative eigenvalues are most strongly affected by modes 1 and 6.
- Using ε=0.9999, after 3 iterations the algorithm converges on:
  - $M_{ts,1}$=0.99824 and $M_{ts,6}$=0.9999
  - (or increase mode scale factors by: 1.0008798 and 1.000050)
- Works perfectly, however, what if we chose ε=0.99?



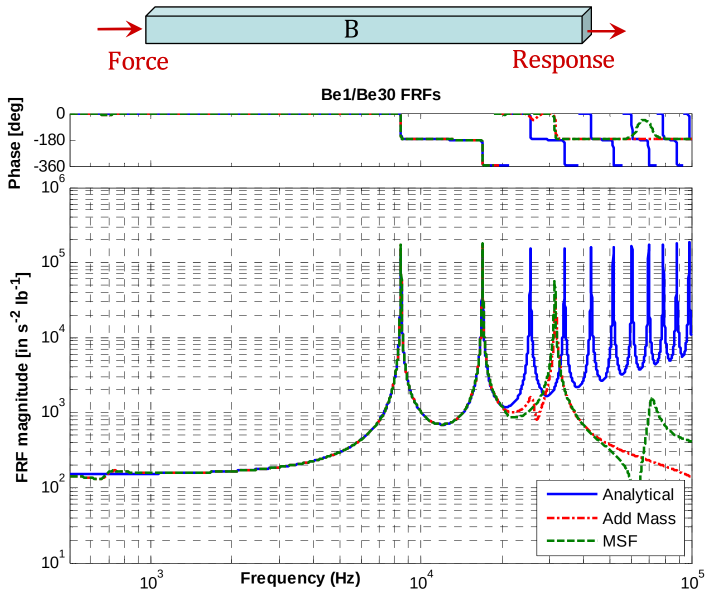
Goal ca… es!

Results: Added Mass Method

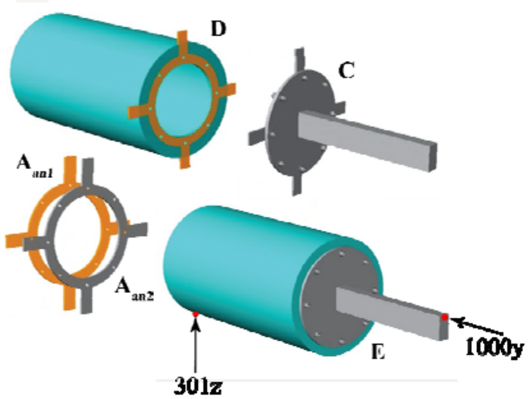
- Using ε=2.2e-14 (100 times larger than machine precision), smallest eigenvalue becomes 2.2e-14 as expected.
- How much mass had to be added to achieve this?
  - Ratio of the norm of $\mathbf{M_B}$ to $\mathbf{\Delta m}$ = 0.000505 (less than 0.05%!)

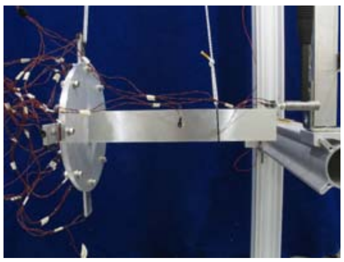


Axial FRFs: Both Methods



Three-Dimensional Plate-Beam System

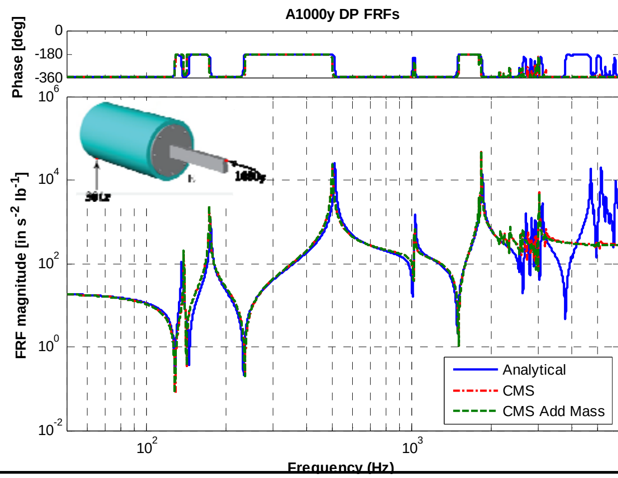




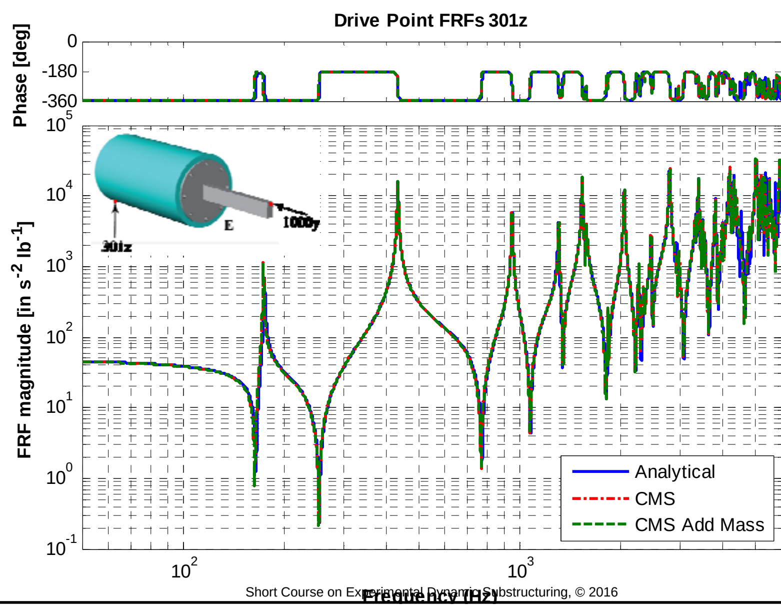
- Test performed to measure the modes of C
- Analytical system D modeled with finite elements with a second copy of the transmission simulator
- C & D are joined and then two copies of the transmission simulator are removed (using the FEA model of the TS)

Case 1: Correct Mass Matrix after Assembling Full System, C+D-2A

- 4 Negative Eig.: λ_1= –0.197, λ_2= –0.0764, λ_3= –0.134, λ_4= –0.118
- Used added mass method, which revealed that 19.7% more mass had to be add to make the mass matrix positive definite.
- No deterioration observed in the quality of the FRFs

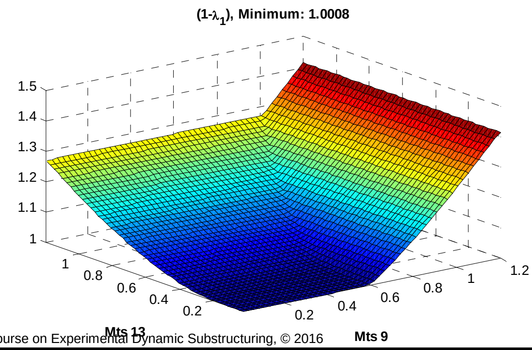


Case 1: Correct Mass Matrix after Assembling Full System, C+D-2A



Case 2: Correct the mass matrix for C-A

- After removing the transmission simulator from C, the resulting model for B has two negative eigenvalues:
  - λ_1=-0.116 and λ_2=-0.0865
  - Using added mass method, the norm of the required mass addition is 11.6% of the norm of M_C
  - Mode Scale Method implemented with the Nelder-Mead Simplex algorithm using the dominant modes found by Kammer's τ metrics, the 9th and 13th modes.
    - Algorithm produced a negative modal mass, which was clearly not reasonable.



Conclusions

- Two methods have been proposed for correcting the negative mass that is sometimes obtained in substructuring predictions.
- One is highly unlikely to find a set of scale factors that produce positive mass using trial and error, so these new algorithms are a significant aid!
- Added Mass Method
  - Is straightforward to implement and was found to be successful in a variety of challenging cases.
  - Sometimes quite a large amount of mass had to be added.
- Modal Scale Factor Method:
  - This approach is more easily justified physically, since it avoids subtracting too much mass rather than correcting negative mass after the fact.
  - A nonlinear optimization problem must be solved to compute the modal scale factors of the transmission simulator that produce the desired result.

Additional Comments by R.L. Mayes